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PHASE-ONLY NULLING AS A LEAST-SQUARES APPROXIMATION TO COMPLEX WEIGHT NULLING

Robert A. Shore

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A solution is presented to the problem of obtaining a least-squares fit of phase-only array weight perturbations to a set of combined phase and amplitude perturbations calculated to impose pattern nulls at prescribed sidelobe directions. It is shown that the desired least-squares fit is accomplished by taking only the phase part of the combined phase and amplitude perturbations. This method of phase-only nulling is compared in effectiveness with other phase-only pattern synthesis methods.

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1. INTRODUCTION

There is currently considerable interest, arising from the widespread use of phased arrays, in phase-only control of array weights for adaptive cancellation of interference[1-5] and for non-adaptive shaping of antenna patterns.[6-8] To develop efficient procedures of adaptive phase-only cancellation, it is useful to have a clear understanding of the end products of such adaptive procedures, namely, the kinds of patterns with imposed nulls that can be obtained with phase-only weight perturbations. Also of interest, because of their potential utility in formulating adaptive procedures, are deterministic methods of phase-only null synthesis. It is for these reasons that we have undertaken a series of related studies to investigate different methods of phase-only null synthesis.

The principal difference between null synthesis with phase-only control and with complex (that is, combined phase and amplitude) control, is that the equations for imposing pattern nulls with phase-only control are non-linear, whereas the null equations for complex weight perturbations are linear. The non-linear equations cannot be solved exactly analytically and hence alternative methods must be found to obtain solutions. In our first study[9] we assumed that the required phase

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(Due to the large number of references cited above, they will not be listed here. See References, page 21.)

perturbations of the element weights are small and under this assumption linearized the phase-only null equations. The solution to the linearized equations is a set of phase perturbations that are reasonable approximations to the exact phase perturbations in certain situations, namely low sidelobes and a small number of imposed nulls. The cancellation effectiveness, however, is not comparable to that achievable with complex perturbations. In our second study[10] we developed an iterative procedure for solving the phase-only null equations based on repeated linearization of the equations. First, the simple linearization method is used to form a pattern with lower power at prescribed locations than the original pattern. Then, the new pattern is in turn taken as the starting point and the linearization procedure applied again to obtain a pattern with still lower values of the power at the prescribed locations. The procedure is repeated until nulls as deep as desired are obtained. This iterative method is remarkably successful in low sidelobe applications and even in applications to patterns with high sidelobes and a small number of imposed null locations, although it fails to work as a general method. When the iterative procedure does converge, the cancellation effectiveness obtained with the phase-only perturbations is almost fully comparable to that obtained with complex perturbations. The principal drawback of phase-only null synthesis per se (that is, with near-exact solutions to the non-linear null equations) is that any nulling of the original pattern is accompanied by a corresponding raising of the pattern at symmetric locations with respect to the mainbeam.

In this report we investigate a third method of phase-only null synthesis: that of fitting phase-only perturbations in a least-squares sense to complex perturbations calculated to impose pattern nulls at a prescribed set of sidelobe directions. We show that such a least-squares fit is accomplished very simply by taking only the phase part of the combined phase and amplitude perturbations and ignoring the amplitude change in the element weights. The method, although extremely simple to implement, does not in general give cancellation effectiveness comparable to even the simple linearized equation method. It does however, give modest cancellation in high sidelobe and/or multiple imposed null applications when one or both of the other two methods fail to work at all.

2. ANALYSIS

We begin by proving the central result of this report: Let w_{on}, $n = 1, 2, \ldots, N$, be a set of linear array weights, and let w_n, $n = 1, 2, \ldots, N$, be a set of weights

10. Shore, R. (1982) An Iterative Phase-Only Nulling Method, RADC-TR-82-49.

derived from the original weights by multiplication by the complex factors $\rho_n e^{j\phi_n}$; that is,

$$w_n = w_{on}\,\rho_n\, e^{j\phi_n}, \qquad n = 1, 2, \ldots, N .$$

Then the set of weights $w_{pn} = w_{on}\, e^{j\delta_n}$ derived from w_{on} by phase-only perturbations that differs least in a least-squares sense from the w_n is

$$w_{pn} = w_{on}\, e^{j\phi_n}, \qquad n = 1, 2, \ldots, N .$$

In other words, the phase-only perturbed weights that differ least from the combined phase-and-amplitude perturbed weights are obtained by setting the phases of the phase-only perturbations equal to the phases of the combined phase-and-amplitude perturbations.

The proof is straightforward. Let E be the sum of the squares of the differences between the phase-only perturbed weights and the phase-and-amplitude perturbed weights. Then

$$\begin{aligned} E &= \sum_{n=1}^{N} |w_n - w_{pn}|^2 \\ &= \sum_{n=1}^{N} |w_{on}\,\rho_n\, e^{j\phi_n} - w_{on}\, e^{j\delta_n}|^2 \\ &= \sum_{n=1}^{N} \left[|w_{on}|^2 \left(\rho_n e^{j\phi_n} - e^{j\delta_n}\right)\left(\rho_n e^{-j\phi_n} - e^{-j\delta_n}\right)\right] . \end{aligned}$$

Differentiating E with respect to any particular one of the δ_n and setting the derivative equal to zero

$$\begin{aligned} \frac{\partial E}{\partial \delta_n} &= |w_{on}|^2 \left[\left(\rho_n e^{j\phi_n} - e^{j\delta_n}\right)\left(j\, e^{-j\delta_n}\right) \right. \\ &\quad \left. + \left(\rho_n e^{-j\phi_n} - e^{-j\delta_n}\right)\left(-j\, e^{j\delta_n}\right)\right] \\ &= |w_{on}|^2 \,(j\,\rho_n) \left[e^{j(\delta_n - \phi_n)} - e^{-j(\delta_n - \phi_n)}\right] \\ &= 2|w_{on}|^2\, \rho_n \sin(\delta_n - \phi_n) \\ &= 0 , \end{aligned}$$

whereupon we obtain

$$\delta_n = \phi_n \pm K\pi \tag{1}$$

for any integer K. In particular we can set K = 0 thus obtaining the desired result.

This result can be equivalently expressed in terms of patterns instead of weights, since the sum of the squares of the array weights equals the mean square of the array pattern, the mean being taken over a full period of the variable $u = \frac{2\pi}{\lambda} d \sin\theta$ (d = interelement spacing, λ = wavelength, θ = pattern angle measured from broadside). Thus let P_o be a linear array pattern and P_1 a pattern derived from P_o by perturbations of the amplitudes and phases of the array weights. Then the pattern P_2 derived from P_o by phase-only perturbations of the array weights that most closely matches P_1 in a least mean square sense is obtained by keeping only the phases of the weight perturbations associated with P_1 and neglecting the amplitude perturbations.

In a previous report[9] we derived expressions for the minimum weight perturbations, both complex and small phase-only, required to impose nulls in a given pattern at a set of M specified sidelobe directions. We showed that the cancellation pattern corresponding to the complex weight perturbations could be interpreted as the sum of M beams of the same shape, one directed at each of the null locations. For small phase-only perturbations, the cancellation pattern was expressible as the sum of M pairs of beams, one member of each pair directed at a null location, and the other, of opposite sign, directed at the symmetric location with respect to the mainbeam.

If we obtain a set of phase-only weight perturbations as a least-squares fit to a given set of complex perturbations that corresponds to a sum of M cancellation beams, one directed at each of M null locations, it is again the case that for small perturbations the phase-only cancellation pattern can be interpreted as the sum of M pairs of beams of opposite sign. For, following Ref. 9, suppose that the original weights are given by

$$w_{on} = a_n e^{-j d_n u_s}, \qquad n = 1, 2, \ldots, N \tag{2}$$

where N is the number of elements, the a_n are the (real) amplitudes of the weights, and we have assumed a uniform progressive phase shift $u_s = \frac{2\pi}{\lambda} d \sin\theta_s$ across the array with d the interelement spacing and θ the pattern angle measured from broadside. The phase reference center is taken to coincide with the center of the array, and the factor d_n in the exponent of Eq. (2) is given by

$$d_n = \frac{N-1}{2} - (n-1), \qquad n = 1, 2, \ldots, N .$$

The amplitudes are assumed to be symmetric with respect to the phase reference center. Let us further suppose that the complex weight perturbations, Δw_n, that we are trying to match in a least-squares sense by phase-only perturbations are given by

$$\Delta w_n = c_n \sum_{m=1}^{M} b_m e^{-j d_n u_m} \tag{3}$$

with the corresponding cancellation pattern

$$\sum_{m=1}^{M} b_m \sum_{n=1}^{N} c_n e^{j d_n (u - u_m)} .$$

In Eq. (3), the coefficients c_n that determine the shape of the cancellation beams are assumed to be real and symmetric with respect to the phase reference center; $u_m = \frac{2\pi}{\lambda} d \sin \theta_m$, $m = 1, 2, \ldots, M$ where θ_m is the pattern location of the m^{th} null and b_m, $m = 1, 2, \ldots, M$, is a real coefficient giving the magnitude and sign of the m^{th} cancellation beam. To obtain the phase-only perturbations, Δw_{pn}, that differ least in a least-squares sense from the Δw_n, we first, following the proof of Eq. (1), express the perturbed weights, $w_n = w_{on} + \Delta w_n$, in the form $w_n = w_{on} \rho_n e^{j\varphi_n}$ so that

$$\begin{aligned}
w_n &= w_{on} + \Delta w_n \\
&= w_{on} \left(1 + \frac{\Delta w_n}{w_{on}} \right) \\
&= w_{on} \left[1 + \frac{c_n}{a_n} \sum_{m=1}^{M} b_m e^{-j d_n (u_m - u_s)} \right] \\
&= w_{on} \left[1 + \left(\alpha_n + j \beta_n \right) \right] \\
&= w_{on} \left[(1 + \alpha_n)^2 + \beta_n^2 \right]^{1/2} e^{j \tan^{-1} \left(\frac{\beta_n}{1 + \alpha_n} \right)}
\end{aligned}$$

where

$$\alpha_n = \frac{c_n}{a_n} \sum_{m=1}^{M} b_m \cos \left[d_n (u_m - u_s) \right]$$

and

$$\beta_n = -\frac{c_n}{a_n} \sum_{m=1}^{M} b_m \sin\left[d_n (u_m - u_s)\right] .$$

The phase-only perturbed weights, w_{pn}, differing least from the combined phase-and-amplitude perturbed weights are then

$$w_{pn} = w_{on}\, e^{j \tan^{-1}\left(\frac{\beta_n}{1+\alpha_n}\right)} .$$

For small combined phase-and-amplitude perturbations, both the α_n and the β_n will be small and we can then approximate the w_{pn} by

$$w_{pn} \approx w_{on} (1 + j\beta_n)$$

so that the phase-only weight perturbations are

$$\begin{aligned}
\Delta w_{pn} &= w_{pn} - w_{on} \\
&\approx -j\, w_{on} \frac{c_n}{a_n} \sum_{m=1}^{M} b_m \sin\left[d_n (u_m - u_s)\right] \\
&= j c_n e^{-j d_n u_s} \sum_{m=1}^{M} b_m \left[\frac{e^{-j d_n (u_m - u_s)} - e^{j d_n (u_m - 2 u_s)}}{2j} \right] \\
&= \frac{c_n}{2} \sum_{m=1}^{M} b_m \left[e^{-j d_n u_m} - e^{j d_n (u_m - 2 u_s)} \right] .
\end{aligned}$$

The corresponding phase-only cancellation pattern is then

$$\begin{aligned}
\Delta F_p(u) &= \sum_{n=1}^{N} \Delta w_{pn}\, e^{j d_n u} \\
&= \sum_{m=1}^{M} b_m \sum_{n=1}^{N} \frac{c_n}{2} \left[e^{-j d_n (u_m - u)} - e^{j d_n (u_m - 2 u_s + u)} \right] \\
&= \sum_{m=1}^{M} b_m \sum_{n=1}^{N} \frac{c_n}{2} \left[e^{j d_n (u - u_m)} - e^{j d_n (u - [u_m - 2\{u_m - u_s\}])} \right]
\end{aligned}$$

and so is, as claimed, the sum of M pairs of beams, one member of each pair directed at a null location, $u = u_m$, and the other member of opposite sign directed at the location $u = u_m - 2(u_m - u_s)$, symmetric with respect to the main beam. This means that if we try to do phase-only nulling by a best least-squares fit of the phase-only perturbed weights to a set of amplitude-and-phase perturbed weights, that give imposed nulls at the desired locations, any cancellation of the original pattern will be accompanied by an increase in the power pattern at directions symmetric (with respect to the main beam) to the null locations. No such increase is present in the pattern corresponding to the combined amplitude-and-phase perturbed weights.

3. NUMERICAL RESULTS AND DISCUSSION

In this section we present the results of computations performed to examine the performance of phase-only nulling as a least squares approximation to complex (that is, combined phase-and-amplitude) nulling. All computations were performed for an array of 41 equispaced isotropic elements with half wavelength interelement spacing. The results presented are of two types. One is the depth of the null at the imposed null location(s), and the other is the cancellation achieved throughout a sector of the pattern. Following our earlier report[9] we define the power cancellation ratio, C, in the sector $\Delta\theta = \theta_1 \leq \theta \leq \theta_2$, to be

$$C = \frac{\max\limits_{\theta \epsilon \Delta\theta} \left[F(\theta)\right]^2}{\max\limits_{\theta \epsilon \Delta\theta} \left[F_o(\theta)\right]^2}$$

where $F_o(\theta)$ is the original pattern and $F(\theta)$ is the perturbed pattern.

In Table 1 we show the depth of null achieved with phase-only perturbations obtained as a least-squares approximation to complex nulling, and compare these null depths to those achieved with phase-only perturbations obtained by solving the linearized null equations (see Ref. 9). For reference we also show the null depths obtained with an iterative method described in Ref. 10 which attempts to find exact solutions of the phase-only nulling equations by a process of repeated linearization. The original weights are for a 40 dB Chebyshev pattern. Following Ref. 9, two types of complex nulling are considered, one in which the sum of the squares of the weight perturbations is minimized, and the other in which the sum of the squares of the weight perturbations relative to the original amplitudes of the weights is minimized. The results presented are for a series of equispaced [in sin (θ)] imposed null locations. First, one imposed null at 15.23° is considered. Then two simultaneous

imposed nulls at 15.23° and 15.78° are considered, and so on up to six simultaneous imposed null locations. It is seen that in general for up to five simultaneous imposed nulls, the depth of null at the imposed null locations obtained with phase-only nulling as a least-squares approximation is considerably shallower than that obtained by solving the linearized phase-only nulling equations. Only for the six null case (when the linearized phase-only solution is almost completely ineffective because of the large perturbations that result from interbeam interference) are deeper nulls obtained with the least-squares approximated phase perturbations. The iterative linearization procedure gives consistently far deeper nulls than either of the other two methods.

In Table 2 we show the cancellation ratios for the sectors corresponding to the sets of imposed null locations used in Table 1. The entries in the i^{th} row of the table are the cancellation ratios obtained by the methods listed across the top of the table for the sector $15.23 \le \theta \le (\theta_2)$ with nulls imposed at 15.23° and at the locations $\theta = (\theta_2)_j$, $j \le i$. We note that, for all except the six null case, the sector cancellation ratios are also shallower for the least-squares approximated phase perturbations than for the perturbations obtained as solutions to the linearized null equations. This is to be expected since effective sector cancellation depends in large part on deep nulls being formed at the imposed locations to "anchor the pattern down" as it were. Since the combined phase-and-amplitude perturbations from which the least-squares approximated phase-only perturbations are derived give patterns with virtually perfect nulls and very effective sector cancellation (see Figures 1 and 2) we are forced to conclude that much of the effectiveness of complex nulling is lost when we keep only the phase part and discard the amplitude component of the perturbations. This loss of effectiveness is not an intrinsic property of phase-only nulling per se, as is clear from the null depths and sector cancellation obtained with the iterative linearization procedure. Rather, it is a deficiency of the particular approximation method.

Tables 3 and 4, similar to Tables 1 and 2 respectively, show null depths and cancellation ratios for a series of imposed null locations in a 20 dB Chebyshev pattern. The overall behavior of the results obtained is similar to that for the 40 dB Chebyshev pattern case. For a small number of imposed nulls, the least-squares approximated phase perturbations give shallower nulls and less effective sector cancellation than do the perturbations obtained as solutions to the linearized phase-only null equations. As the number of imposed nulls increases, the linearized phase-only perturbation method breaks down because large phase perturbations are required to impose nulls and hence the small phase perturbation assumption used to obtain the linearized phase-only null equations becomes invalid. (In general, the 20 dB Chebyshev taper pattern requires larger phase perturbations to impose nulls than does the 40 dB pattern because the pattern must be lowered further (that is,

perturbed more) to form deep nulls.) The iterative method, the first iteration of which coincides with the solution of the linearized null equations, also breaks down and further iterations fail to improve on the initial iteration. The least-squares approximated phase perturbations, especially those based on the complex nulling method that minimizes $\sum |\Delta w_n|^2$, continues to give some cancellation for five or six imposed nulls, but the effectiveness of nulling is not impressive and is, in general, less than the effectiveness of nulling shown for the 40 dB case.

4. CONCLUSIONS

In this report we have investigated a method of phase-only nulling in linear array patterns. The method consists of obtaining the set of phase-only weight perturbations that differs least in a least-squares sense from a set of complex (that is, combined phase and amplitude) perturbations calculated analytically to impose nulls at prescribed locations in a given pattern. We showed that the desired phase perturbations are given by taking the phase part only of the complex perturbations and ignoring the change in amplitude of the element weights.

Calculations were performed to examine the effectiveness of this nulling method and to compare it with two other phase-only methods. One of these is based on solving a set of linearized equations obtained from the non-linear phase-only null equations under the assumption of small phase perturbations. The other attempts to find exact solutions of the non-linear null equations by a process of iterative linearization. It was found that the least-squares approximated phase-only perturbations did not give deep nulls at the desired pattern locations and, for a small number of imposed nulls, was less effective than the linearized phase-only perturbation method, which in turn is usually considerably less effective than the iterative procedure. However, the least-squares approximated phase method did provide some cancellation in situations where one or both of the other two methods failed to work at all; namely, patterns with high sidelobes and/or many imposed null locations.

Table 1. Depth of Null (dB) at Imposed Null Directions θ_{null}; 40 dB Chebyshev Amplitude Taper

	Phase-Only Perturbations Obtained as a Least-Squares Approximation to Complex Nulling Solution		Phase-Only Perturbations Obtained by Solving the Linearized Null Equations		Phase-Only Perturbations Obtained by Iterative Linearization Method	
θ_{null} (deg)	$\sum \lvert\Delta w_n\rvert^2 = \min$	$\sum (\lvert\Delta w_n\rvert / a_n)^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$
15.23	-46	-46	-97	-124	< -260	< -260
15.23	-46	-46	-87	-121	< -260	< -260
15.78	-47	-48	-84	-123	< -260	< -260
15.23	-46	-46	-67	-87	< -260	< -260
15.78	-49	-48	-68	-111	< -260	< -260
16.33	-61	-58	-73	-86	< -260	< -260
15.23	-45	-45	-54	-95	< -260	-251
15.78	-48	-48	-52	-108	< -260	< -260
16.33	-60	-61	-52	-95	< -260	-260
16.88	-53	-52	-56	-86	< -260	< -260
15.23	-47	-47	-53	-72	< -260	< -260
15.78	-51	-51	-52	-89	< -260	-257
16.33	-78	-75	-54	-76	< -260	< -260
16.88	-51	-52	-60	-71	< -260	-254
17.44	-47	-47	-77	-70	< -260	< -260
15.23	-55	-58	-41	-42	-249	-230
15.78	-75	-60	-43	-41	< -260	-221
16.33	-56	-50	-52	-43	-238	-232
16.88	-51	-47	-52	-46	< -260	-230
17.44	-51	-47	-44	-51	-257	-228
18.00	-54	-52	-41	-63	-246	-227

Table 2. Values of the Cancellation Ratio (dB) for the Sector $15.23° \leq \theta \leq \theta_2$ With Equispaced Imposed Nulls and a 40 dB Chebyshev Amplitude Taper

	Phase-Only Perturbations Obtained as a Least-Squares Approximation to Complex Nulling Solution		Phase-Only Perturbations Obtained by Solving the Linearized Null Equation		Phase-Only Perturbations Obtained by Iterative Linearization Method	
θ_2 (deg)	$\sum\lvert\Delta w_n\rvert^2 = \min$	$\sum(\lvert\Delta w_n\rvert/a_n)^2 = \min$	$\sum(a_n\phi_n)^2 = \min$	$\sum\phi_n^2 = \min$	$\sum(a_n\phi_n)^2 = \min$	$\sum\phi_n^2 = \min$
15.78	-5.5	-5.7	-31.4	-27.8	-29.8	-27.8
16.33	-6.0	-6.2	-27.3	-45.5	-46.6	-43.1
16.88	-5.4	-5.3	-11.4	-51.8	-62.9	-56.7
17.44	-6.7	-6.8	-12.2	-29.5	-68.0	-62.5
18.00	-10.3	-6.6	-0.7	-1.3	-82.0	-77.9

Table 3. Depth of Null (dB) at Imposed Null Directions, θ_{null}; 20 dB Chebyshev Amplitude Taper

	Phase-Only Perturbations Obtained as a Least-Squares Approximation to Complex Nulling Solution		Phase-Only Perturbations Obtained by Solving the Linearized Null Equations		Phase-Only Perturbations Obtained by Iterative Linearization Method	
θ_{null} (deg)	$\sum \lvert \Delta w_n \rvert^2 = \min$	$\sum (\lvert \Delta w_n \rvert / a_n)^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$
14.70	-26	-25	-64	-80	< -260	< -260
14.70	-26	-24	-91	-92	< -260	< -260
15.28	-27	-27	-57	-79	< -260	< -260
14.70	-27	-26	-34	-40	-54	< -260
15.28	-29	-38	-73	-42	-59	< -260
15.86	-37	-30	-35	-52	-85	< -260
14.70	-26	-25	-29	-29	-29	-29
15.28	-28	-37	-33	-65	-33	-65
15.86	-40	-29	-23	-30	-23	-30
16.44	-33	-23	-20	-26	-20	-26
14.70	-28	-29	-27	-29	-27	-29
15.28	-34	-39	-33	-43	-33	-43
15.86	-41	-25	-22	-27	-22	-27
16.44	-29	-22	-19	-24	-19	-24
17.03	-26	-23	-19	-24	-19	-24
14.70	-28	-43	-20	-36	-20	-36
15.28	-41	-25	-40	-21	-40	-21
15.86	[illegible]	-22	-21	-15	-21	-15
16.44	-25	-22	-16	-13	-16	-13
17.03	-24	-26	-16	-13	-16	-13
17.62	-27	-60	-18	-17	-18	-17

Table 4. Values of the Cancellation Ratio (dB) for the Sector $14.70° \leq \theta \leq \theta_2$ With Equispaced Imposed Nulls and a 20 dB Chebyshev Amplitude Taper

	Phase-Only Perturbations Obtained as a Least-Squares Approximation to Complex Nulling Solution		Phase-Only Perturbations Obtained by Solving the Linearized Null Equation		Phase-Only Perturbations Obtained by Iterative Linearization Method	
θ_2 (deg)	$\sum \lvert \Delta w_n \rvert^2 = \min$	$\sum (\lvert \Delta w_n \rvert a_n)^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$	$\sum (a_n \phi_n)^2 = \min$	$\sum \phi_n^2 = \min$
15.28	-5.6	-3.9	-32	-28	-30	-28
15.86	-6.9	-5.8	-14	-20	-33	-43
16.44	-5.6	-2.3	0	-6	0	-6
17.03	-5.9	-1.8	+1	-3	+1	-3
17.62	-3.8	-1.4	+5	+7	+5	+7

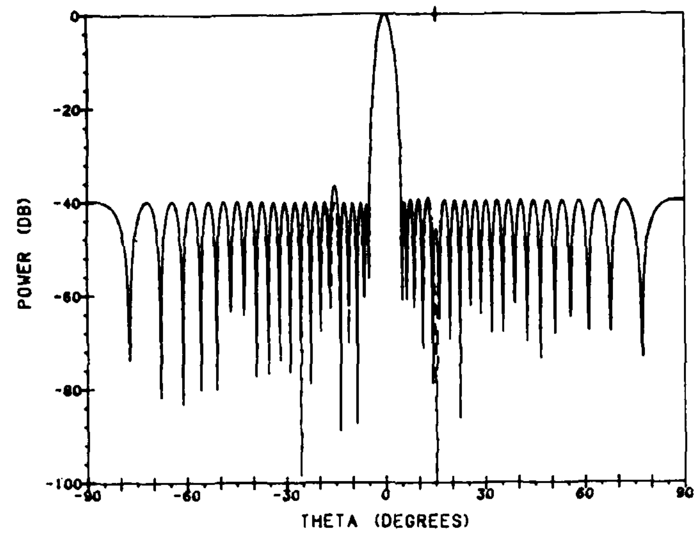

Figure 1a. Perturbed Patterns Obtained by Complex Weight Perturbations (----) and Least-squares Approximated Phase-only Perturbations (——) With One Imposed Null at 15.23°. Range of θ: [-90°, 90°]

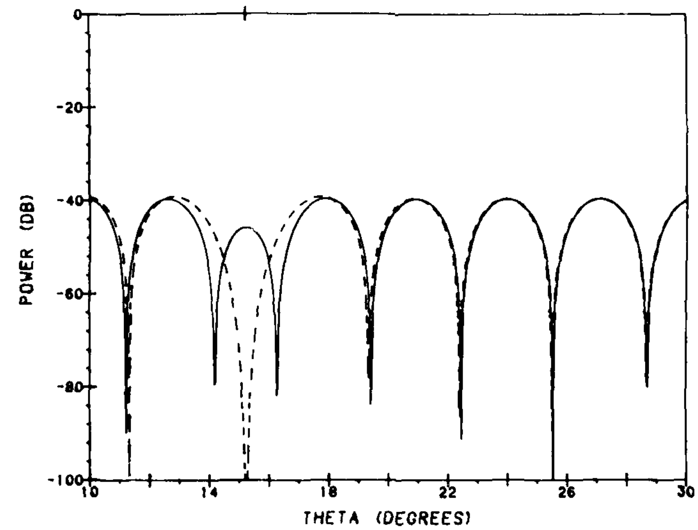

Figure 1b. Perturbed Patterns Obtained by Complex Weight Perturbations (----) and Least-squares Approximated Phase-only Perturbations (——) With One Imposed Null at 15.23°. Range of θ: [10°, 30°)

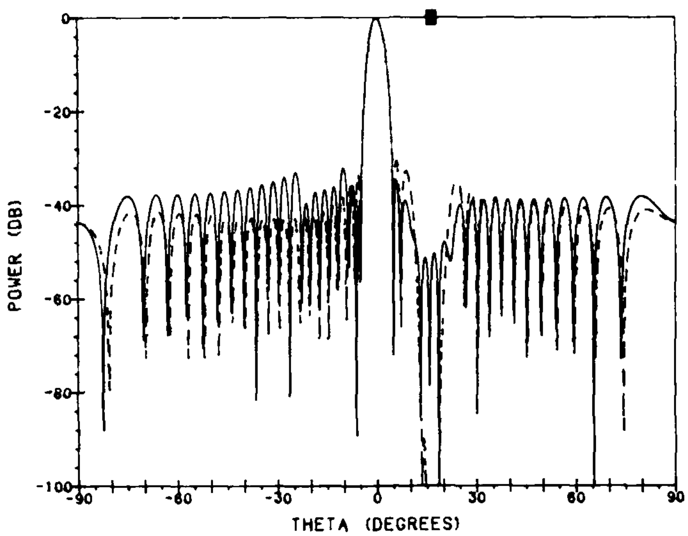

Figure 2a. Perturbed Patterns Obtained by Complex Weight Perturbations (----) and Least-squares Approximated Phase-only Perturbations (——) With Six Imposed Nulls at 15.23°, 15.78°, 16.33°, 16.88°, 17.44°, and 18.00°. Range of θ: [-90°, 90°)

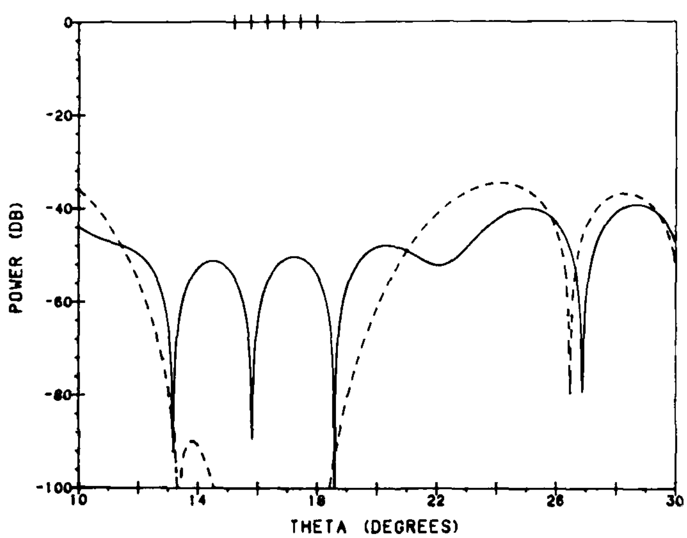

Figure 2b. Perturbed Patterns Obtained by Complex Weight Perturbations (----) and Least-squares Approximated Phase-only Perturbations (——) With Six Imposed Nulls at 15.23°, 15.78°, 16.33°, 16.88°, 17.44°, and 18.00°. Range of θ: [10°, 30°]

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